Electromagnetic Energy Flow in Photonic Crystals

Photonic crystals (PCs) are periodic dielectric structures that enable an efficient control of optical electromagnetic signals (light) in geometries with features on the sub-wavelength scale; a control that is not possible in classical optics based on total internal reflection and refraction of light rays. In photonic crystals a periodic photonic potential can induce a photonic bandgap, i.e.; a range of frequencies where radiation energy in certain polarisation states is not allowed to flow in specific directions. This is the photonic analogue of the forbidden energy band for electrons in an electric periodic potential in the lattice of atoms of a semiconductor material.

Photonic crystals enable new advanced all-optical signal processing in regions with dimensions of a few cubic wavelengths because radiation losses can be greatly reduced; something that is impossible in classical optical components without serious energy loss.

Electromagnetic phenomena are governed by Maxwell s equations and a set of boundary conditions that the electric vector field **E** and the magnetic vector field **H** must satisfy at material interfaces.

Figure 1 shows a planar cut of a two-dimensional (2D) photonic crystal made of a background material of relative dielectric constant ε_{r1} in which a triangular lattice of holes is etched. For silicon nitride the relative dielectric constant is assumed to be 4 at the wavelengths of interest, and that of air is 1. The lattice constant is Λ and the hole radius is r0. The eigenstates of this 2D structure can be categorised as H states (the magnetic field vector is parallel to the 2D plane and the electric field vector is perpendicular to the same) or E states (the electric field vector is parallel to the 2D plane and the magnetic field vector is perpendicular to the same). Banddiagrams usually present eigenfrequencies, $\omega=\Lambda/\lambda$, versus **k** vector values, where λ is the free-space wavelength and **k** is the Bloch mode propagation vector. The chosen **k** vectors usually lie on the contour of the irreducible Brillouin zone in order to determine the existence of bandgaps (the smallest bandgap is determined by the highest photon energies). The preferred regime of operation is typically for $\omega<1$.

We assume that only E states are present in the configuration.

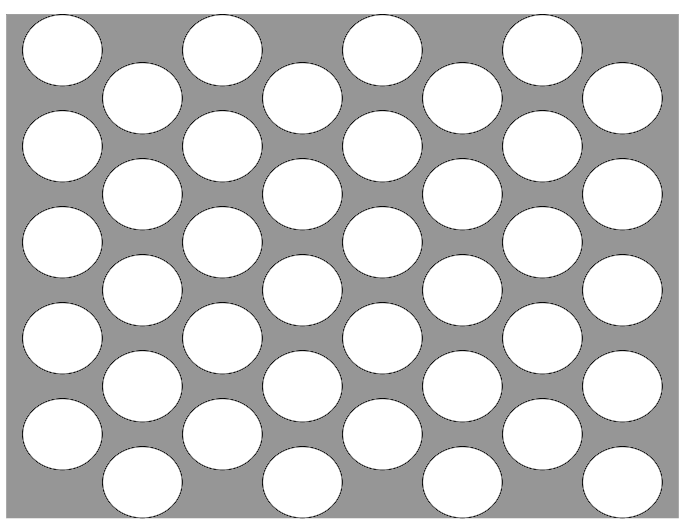


Figure 1: Two-dimensional photonic crystal made of a background material in which a triangular lattice of holes is etched.

By creating line defects of width W along the nearest neighbour direction, we can introduce localised electromagnetic states $\omega_d(\mathbf{k})$ with frequencies in the bandgap of the perfect crystal (for any crystal configuration exhibiting a gap of course).

Some of the questions one wonders about:

Eigenstates:

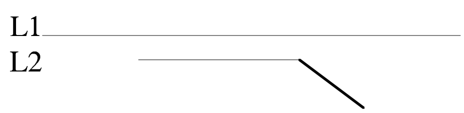
- For any given **k** there are infinitely many eigenfrequencies $\omega(\mathbf{k},n)$ for n = 1,2, For each eigenfrequency $\omega(\mathbf{k},n)$ we have the eigenvectors $\mathbf{H}(\mathbf{k},n)$ and $\mathbf{E}(\mathbf{k},n)$ (only one of these need to be determined). We often only are interested in solutions for which $\omega<1$. Is there an analytical solution for the perfect PC and the PC with line defects ?
- Assuming that we have the eigenstates of the perfect PC up to n=N analytically or numerically, is there a rather simple way of finding the eigenstates $(\omega(\mathbf{k},n), \mathbf{F}(\mathbf{k},n))$, $\mathbf{F} = \mathbf{H}$ or $\mathbf{E}$, of the PC with line defects ?
- The material processing induces small deformations where the lattice constant is rather constant , but where the shape of the holes varies a little in the crystal. This variation concerns the diameter of the holes. The largest diameter variation is about 15% (+/-7.5 %)of the mean diameter. What is the sensitivity of the bandgap to a normal distribution of diameter variations in the crystal without line defects ? and what is the sensitivity of the localised state $\omega_d(\mathbf{k})$ to this variation in a crystal with a line defect made of holes with mean diameter D and a defect hole variation of 15% ?

Radiation:

- Assume that we have a finite crystal of depth d in one dimension and infinite in the other two, and two regions with homogeneous materials above it and below it. The first region above has a relative dielectric constant ε_{r1} and the other one below a relative dielectric constant ε_{r2}. Assuming that a generally polarised plane wave of specific wavelength is incident upon the crystal from region 1, can we derive expressions for the reflected field in region 1, and the transmitted field in region 2 ?

Energy coupling:

- Assume that two line defects, L1 and L2, are introduced in the perfect 2D crystal. L1 guides energy in a defect state $\omega_d(\mathbf{k})$ with different wavelengths. L1 and L2 are parallel to each other separated by one or more row(s) of holes. What is the maximal coupling efficiency if we try to dump one wavelength signal into L2 ? What is the minimal coupling length, and the minimal line width of the dumped signal (wavelength span relative to the wavelength) for coupling efficiencies above 80% taking into account the diameter variation of max 15% ?



Negative refraction:

- Photonic crystals might enable focusing below the classical diffraction limit (imaging of point sources) see f.ex. [C. Luo, S.G. Johnson, J.D. Joannopoulos, and J.B. Pendry, All-angle negative refraction without negative effective index , Phys. Rev. B, Vol. 65, 201104 (2002)]. Is this superlensing effect possible for ω values closer to 0.5 if the background has a relative dielectric constant of 2.1316 ? and what is the maximal bandwidth for which superlensing is achievable ?

The numerical investigations are often based on methods mentioned in the following publications:

- S. G. Johnson and J. D. Joannopoulos, "Block-iterative frequency-domain methods for Maxwell's equations in a planewave basis," *Optics Express* **8**, no. 3, pp. 173-190 (Jan. 2001).

- R. D. Meade, A. M. Rappe, K. D. Brommer, J. D. Joannopoulos, and O. L. Alerhand, "Accurate theoretical analysis of photonic band-gap materials," *Phys. Rev. B* **48**, no. 11, pp. 8434-8437 (Sep 1993). (abstract)
  - Erratum by S. G. Johnson, *PRB* **55**, no. 23, pp. 15942 (June 1997).

- Steven G. Johnson, M. Ibanescu, M. A. Skorobogatiy, O. Weisberg, J. D. Joannopoulos, and Y. Fink, "Perturbation theory for Maxwell's equations with shifting material boundaries," *Phys. Rev. E* **65**, 066611 (2002).

- A. Mekis, S. Fan, J. D. Joannopoulos, "Absorbing boundary conditions for FDTD simulations of photonic crystal waveguides," *IEEE Microwave and Guided Wave Letters* **9**, p. 502 (1999).

Related literature:

- J.D. Joannopoulos, R.D. Meade, and J.N. Winn, Photonic Crystals: Molding the flow of light , Princeton University Press, 1995.
- T. S¿ndergaard and K.H. Dridi, "Energy flow in photonic crystal waveguides", Physical Review B, Vol. 61, No. 23, June 15 2000, pp. 15688-15696.
- S. G. Johnson, S. Fan, P. R. Villeneuve, J. D. Joannopoulos, and L. A. Kolodziejski, ''Guided modes in photonic crystal slabs'', Physical Review B, Vol. 60, No. 8, pp. 5751-5758 (1999).
- S. G. Johnson, P. R. Villeneuve, S. Fan, and J. D. Joannopoulos, ''Linear waveguides in photonic-crystal slabs'', Physical Review B, Vol. 62, No. 12, pp. 8212-8222 (2000).
- C. Luo, S.G. Johnson, J.D. Joannopoulos, and J.B. Pendry, All-angle negative refraction without negative effective index , Phys. Rev. B, Vol. 65, 201104 (2002)